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# On solving Schwinger-Dyson equations for non-Abelian gauge theory 

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#### Abstract

A method for solving the Schwinger-Dyson equations for the Green function generating functional of non-Abelian gauge theory is proposed. The method is based on an approximation of the Schwinger-Dyson equations by exactly soluble equations. For the $S U(2)$ model the first-step equations of the iteration scheme are solved which define a gauge field propagator. Apart from the usual perturbative solution, a non-perturbative solution is found, which corresponds to the spontaneous symmetry breaking and obeys the infrared finite behaviour of the propagator.


## 1. Introduction

The Schwinger-Dyson equation (SDE) method is one of the basic tools for investigating of the Green functions of quantum theory. Hitherto the one universal method for solving SDEs has been the coupling constant perturbation theory (hereafter simply termed perturbation theory). The range of applicability of other methods (for example, the $1 / N$ expansion) is limited by a narrow class of models. In particular, the $1 / N$-expansion method cannot be used in the calculations for non-Abelian gauge theories due to the complicated structure of the leading approximation.

On the other hand, the applicability of perturbation theory to the investigation of nonAbelian gauge theories is limited by a deep-Euclidean region. In the non-perturbative region of small momenta the physical vacuum of non-Abelian gauge theories obeys a non-trivial structure which is beyond the framework of perturbation theory. In SDE terms this fact can be understood if one takes into account the radical difference in the properties of the leading-approximation equations of the perturbation theory and the original exact equations. The SDEs for the generating functional of the Green functions are equations in functional derivatives. The leading approximation of perturbation theory comprises neglecting the higher-derivative terms in these equations (just such terms correspond to an interaction). The leading-approximation equations of perturbation theory are of a lower order compared with the exact ones; therefore, the class of solutions thus described contracts drastically, and non-perturbative solutions which correspond to the non-trivial physical vacuum practically drop out of consideration. This feature of SDEs in the non-perturbative region is noted repeatedly in simple models (see [1]).

In this work a method for the SDE solution of non-Abelian gauge theory is proposed. It takes into account the terms with higher derivatives (i.e. self-interaction of the non-Abelian

[^0]fields) $a b$ ovo in the leading approximation. Though we limit ourselves to the simplest extension of the class of SDE solutions, the results are non-trivial: the non-perturbative solution which corresponds to spontaneous symmetry breaking (with non-Higgs mechanism) is found. It obeys a non-singular behaviour of the propagator in the non-perturbative infrared region of small momenta.

An idea of the method comprises the approximation of SDEs for the generating functional by equations with 'constant' (i.e. independent of sources) coefficients. These equations have a simple exponential solution, which is a foundation for the linear iteration scheme. The method is universal, as is perturbation theory, i.e. it is applicable to practically any model of quantum field theory. For the scalar $\phi^{4}$ theory it has been shown in [2] that the method describes such non-perturbative phenomena as spontaneous symmetry breaking and the trivialization of $\phi^{4}$ theory in $d=4$. This method has also been successfully applied in the investigation of the Gross-Neveu model for finite $N$ [3].

## 2. Schwinger-Dyson equations and the iteration scheme

A system of SDEs for the generating functional $G(J, \eta)$ of the Green functions of nonAbelian gauge theory has the form
$\mathcal{D}_{\nu}^{a b}\left(\frac{\delta}{i \delta J}\right) F_{\nu \mu}^{b}\left(\frac{\delta}{i \delta J}\right) G+\frac{1}{\alpha} \partial_{\mu} \partial_{\nu} \frac{\delta G}{i \delta J_{v}^{a}}+g f^{a b c} \frac{\delta}{\delta \bar{\eta}^{c}} \partial_{\mu} \frac{\delta G}{\delta \eta^{b}}+J_{\mu}^{a} G=0$
$i \partial_{\mu} \mathcal{D}_{\mu}^{a b}\left(\frac{\delta}{i \delta J}\right) \frac{\delta G}{\delta \bar{\eta}^{b}}+\eta^{a} G=0$.
Here $F_{\mu \nu}^{a}(A)=\partial_{\mu} A_{\nu}^{a}-\partial_{\nu} A_{\mu}^{a}+g f^{a b c} A_{\mu}^{b} A_{\nu}^{c}$ is the gauge field tensor, $\mathcal{D}_{\mu}^{a b}(A)=\delta^{a b} \partial_{\mu}-$ $g f^{a b c} A_{\mu}^{c}$ is the covariant derivative, $f^{a b c}$ are structure constants of a gauge group, $J_{\mu}^{a}(x)$ is the source of the gauge field, $\eta^{a}(x)$ is the source of a ghost field, $\alpha$ is the gauge parameter, and $g$ is the coupling constant. We work in Minkowski space with a metric $(1,-1,-1,-1)$, and $x_{\mu} y_{\mu} \equiv g_{\mu \nu} x^{\mu} y^{\nu}$ by definition.

At $g=0$ the system (1), (2) has a solution

$$
G_{\mathrm{frre}}=\exp \left\{\frac{1}{2 i} J * D_{\mathrm{free}} * J+i \bar{\eta} * \Delta_{\mathrm{free}} * \eta\right\}
$$

where $D_{\text {free }}$ and $\Delta_{\text {free }}$ are the free propagators of the gauge fields and ghost field, respectively, and

$$
J * D_{\mathrm{free}} * J \equiv \int \mathrm{~d} x \mathrm{~d} y J_{\mu}^{a}(x)\left(D_{\mathrm{free}}\right)_{\mu \nu}^{a b}(x-y) J_{\nu}^{a}(y)
$$

etc. The functional $G_{\text {free }}$ is the generating functional of the free Green functions. The iteration scheme based on $G_{\text {free }}$ is the usual coupling constant perturbation theory.

We shall use an alternative iteration scheme for the SDEs (1), (2), which is formulated as follows: the leading approximation is a system of equations with terms proportional to the sources $J$ and $\eta$ omitted. (For the system of equation (1), (2) these are the last terms.) This system has a solution

$$
G_{0}=\exp i\{J * V+\bar{\eta} * C+\bar{C} * \eta\}
$$

where $J * V \equiv \int \mathrm{~d} x J_{\mu}^{a}(x) V_{\mu}^{a}(x)$, etc. The coefficient functions $V_{\mu}^{a}$ and $C^{a}$ are solutions of the corresponding system of 'characteristic equations'. When constructing the iteration scheme for the generating functional

$$
G=G_{0}+G_{1}+\cdots+G_{n}+\cdots
$$

the omitted terms $J G$ and $\eta G$ should be considered as perturbations, i.e. the equations of the iteration scheme are

$$
\begin{align*}
& \left\{\mathcal{D}_{\nu}\left(\frac{\delta}{i \delta J}\right) F_{\nu \mu}\left(\frac{\delta}{i \delta J}\right)+\frac{1}{\alpha} \partial_{\mu} \partial_{\nu} \frac{\delta}{i \delta J_{\nu}}+g f \frac{\delta}{\delta \bar{\eta}} \partial_{\mu} \frac{\delta}{\delta \eta}\right\} G_{n}=-J_{\mu} G_{n-1}  \tag{3}\\
& i \partial_{\mu} \mathcal{D}_{\mu}\left(\frac{\delta}{i \delta J}\right) \frac{\delta G_{n}}{\delta \bar{\eta}}=-\eta G_{n-1} . \tag{4}
\end{align*}
$$

A solution of equations (3), (4) has the form $G_{n}=P_{n} G_{0}$, where $P_{n}$ is a polynomial in $J$ and $\eta$. Therefore, at each iteration step we obtain a closed system of equations for the coefficient functions of the polynomial $P_{n}$, which completely defines the Green functions of the given step. There is no manifest small parameter in the usual sense in this scheme: 'smallness' is defined by the condition that the Green functions are derivatives of the generating functional at $J=\eta=0$, and it is sufficient for us to know $G(J, \eta)$ near zero, i.e. in the region where the neglected terms are small. At each step of the iteration we approximate the functional $G / G_{0}$ by a sum of the polynomials $P_{n}$, and the degree of the polynomial increases with each step. As is known, for ordinary differential equations the scheme of this type is equivalent to iterations of Volterra-type integral equations and gives an expansion that converges well. That is why we may hope that this scheme possesses good convergence properties. In any case it is clear that the convergence of this scheme is no worse than that of the perturbation theory. The perturbation theory is singular in the sense of differential equation theory, since the higher derivatives are omitted in the leading approximation. In contrast to the singular perturbation theory, the scheme proposed is regular in the above sense. This circumstance gives us hope for improving the convergence properties (see [4] for further discussion).

To remove the ultraviolet divergences it is necessary to supplement the SDEs (1), (2) and the iteration scheme equations (3), (4) with the corresponding counterterms. The counterterms are also defined by the iteration procedure: $\delta z=\delta z_{0}+\delta z_{1}+\cdots$, i.e. at each step it is necessary to take into account the counterterms of the corresponding order.

Let us consider the leading approximation in more detail. As has been noted above, the solution of the leading approximation equations is the linear exponential in the sources. The vacuum structure is defined by the solutions of the characteristic equations for $V_{\mu}^{a}$ and $C^{a}$ :

$$
\begin{align*}
& \mathcal{D}_{\nu}(V) F_{\nu \mu}(V)+\frac{1}{\alpha} \partial_{\mu} \partial_{\nu} V_{\nu}+g f C \partial_{\mu} \bar{C}=0  \tag{5}\\
& \partial_{\mu} \mathcal{D}_{\mu}(V) C=0 .
\end{align*}
$$

These equations have a great number of solutions, which reflects the non-trivial vacuum structure of non-Abelian gauge theory. In this connection it is appropriate to recall 't Hooft's conjecture [5] on the existence of different vacuum modes for non-Abelian gauge theory at zero temperature: a superconducting one (spontaneously broken), dual superconducting (confinement), etc. Realization of either mode depends on the values of some quantities (manager parameters) whose definition is a problem of the quantum field dynamics. It is reasonable to suppose that the whole set of solutions of the characteristic equations (5) is divided into classes, each corresponding to some mode of the gauge theory in the above sense.

In this paper we limit ourselves to investigation of the first-step of the iteration scheme based on the simplest solutions of the characteristic equations (5).

The trivial solution of equations (5), $V_{\mu}^{a}=C^{a}=0$ (i.e. $G_{0}=1$ ), leads to an iteration scheme which is simply the reconstructed perturbation theory. The simplest extension of
the set of vacuum solutions compared with the trivial perturbative vacuum comprises taking into account non-zero constant solutions of equations (5). Namely, we choose the leading approximation in the form

$$
\begin{equation*}
G_{0}=\exp i\{J * V\} \tag{6}
\end{equation*}
$$

where the vacuum vectors $V_{\mu}^{a}$ are independent of the spacetime variables and satisfy the condition

$$
\begin{equation*}
f^{a b c} f^{c d h} V_{v}^{b} V_{v}^{d} V_{\mu}^{h}=0 \tag{7}
\end{equation*}
$$

which follows from the characteristic equations (5), if $C=\partial V=0$. Below we shall consider only this class of the leading approximation solutions. Surely we have no prior physical foundation (except for reasons of simplicity) for the choice of this class of solutions. Nevertheless, we shall see that this simple set of solutions leads to a new spontaneously broken mode.

Concluding our discussion of the leading approximation, let us touch upon the definition of the ground state ('physical vacuum'). Due to the linearity of the SDEs (1), (2) and the iteration scheme equations (3), (4), an arbitrary linear combination of solutions is also a solution. Therefore, the physical vacuum functional $G=\langle 0 \mid 0\rangle_{J}$ should be constructed as a linear combination of partial solutions, each of them corresponding to some solution of the characteristic equations (5). In other words, the physical vacuum is a superposition of $V$ vacua. This combination should be chosen in such a way as to maintain the admissible physical properties of the Green functions (for example, $\langle 0| A_{\mu}^{a}|0\rangle=0$ ) and the energy minimality condition.

To formulate the iteration scheme in terms of the polynomials $P_{n}$, it is convenient to introduce the matrix quantity

$$
\begin{equation*}
W_{\mu}^{a b}=i g f^{a b c} V_{\mu}^{c} \tag{8}
\end{equation*}
$$

Then, the iteration scheme equations have the form of equations for the polynomials $P_{n}$ :

$$
\begin{align*}
& \left\{\left[\mathcal{D}_{\nu}(V)+i g f \frac{\delta}{\delta J_{v}}\right]\left[F_{\nu \mu}\left(\frac{\delta}{i \delta J}\right)+W_{v} \frac{\delta}{\delta J_{\mu}}-W_{\mu} \frac{\delta}{\delta J_{v}}\right]\right. \\
& \left.+i\left[W_{\mu}, W_{\nu}\right] \frac{\delta}{\delta J_{v}}+\frac{1}{\alpha} \partial_{\mu} \partial_{v} \frac{\delta}{i \delta J_{v}}+g f \frac{\delta}{\delta \bar{\eta}} \partial_{\mu} \frac{\delta}{\delta \eta}\right\} P_{n}=-J_{\mu} P_{n-1}  \tag{9}\\
& \partial_{\mu}\left[\mathcal{D}_{\mu}(V)+i g f \frac{\delta}{\delta J_{\mu}}\right] \frac{\delta P_{n}}{\delta \bar{\eta}}=i \eta P_{n-1} . \tag{10}
\end{align*}
$$

Here $P_{0}=1$. The first-step solution $P_{1}$ defines the leading approximation for the propagators of the gauge and ghost fields. The following steps define the many-particle functions.

## 3. Solution of first-step equations

The solution of the first-step equations of the iteration scheme (9), (10) is the quadratic polynomial in the sources

$$
\begin{equation*}
P_{1}=\frac{1}{2 i} J * D * J+i \bar{\eta} * \Delta * \eta \tag{11}
\end{equation*}
$$

Equations (9), (10) give us equations for the functions $D_{\mu \nu}^{a b}(x-y)$ and $\Delta^{a b}(x-y)$. The equation for $D_{\mu \nu}$ can essentially be simplified by modifying the gauge condition. Instead of
the usual covariant gauge used above, it is convenient to use the modified gauge condition (' $V$-gauge') with the gauge-fixing term

$$
\begin{equation*}
\mathcal{L}_{\text {gauge }}=-\frac{1}{2 \alpha}\left(\mathcal{D}_{\mu}(V) A_{\mu}\right)^{2} \tag{12}
\end{equation*}
$$

The ghost terms should be changed correspondingly. For the transition to the $V$-gauge in equations (1)-(4), (5), (9), (10), it is sufficient to perform the substitution

$$
\begin{equation*}
\partial_{\mu} \rightarrow \mathcal{D}_{\mu}(V) \tag{13}
\end{equation*}
$$

An essential prerequisite is that the leading approximation condition (7) does not change in the case in question.

The equations have a particularly simple form in the gauge $\alpha=1$ ('diagonal $V$-gauge'). Then, the equation for $D_{\mu \nu}$ in momentum space is

$$
\begin{equation*}
\left\{\mathcal{K}^{2} g_{\mu \nu}+2\left[\mathcal{K}_{\mu}, \mathcal{K}_{\nu}\right]\right\} \tilde{D}_{\nu \lambda}(k)=-g_{\mu \lambda} \tag{14}
\end{equation*}
$$

where $k$ is the momentum, and the following notation has been introduced:

$$
\begin{equation*}
\mathcal{K}_{\mu}^{a b}=k_{\mu} \delta^{a b}-W_{\mu}^{a b} \tag{15}
\end{equation*}
$$

In the region of large $k$ equation (14) tends to the equation for the free propagator in the diagonal gauge, i.e. at $k \rightarrow \infty$

$$
\begin{equation*}
\tilde{D}_{\mu \nu}(k) \approx-\frac{1}{k^{2}} g_{\mu \nu} \tag{16}
\end{equation*}
$$

The equation for the ghost propagator $\Delta$ is

$$
\begin{equation*}
\mathcal{K}^{2} \tilde{\Delta}(k)=-1 \tag{17}
\end{equation*}
$$

In the large- $k$ region the propagator $\tilde{\Delta}$ also tends to the free propagator. Therefore, the ultraviolet behaviour of the solutions with a non-trivial vacuum vector $\boldsymbol{V}_{\mu}$ is the same as for the usual perturbation theory.

Below we consider the case of $S U(2)$ gauge group and restrict ourselves to the subset of constant $V_{\mu}$ with zero field tensor $F_{\mu \nu}(V)=0$, or, equivalently, with

$$
\begin{equation*}
\epsilon^{a b c} V_{\mu}^{b} V_{v}^{c}=0 \tag{18}
\end{equation*}
$$

It is clear that condition (18) ensures the leading-approximation condition (7). The energy of that type of solution is zero, as for the perturbative solution with trivial vacuum. In that sense, this set of solutions can be termed quasiperturbative. Equation (18) gives us $\left[\mathcal{K}_{\mu}, \mathcal{K}_{\nu}\right]=\left[W_{\mu}, W_{\nu}\right]=0$, and the solution of equation (14) is reduced to the inversion of the matrix $\mathcal{K}^{2}$. The result is

$$
\begin{equation*}
\tilde{D}_{\mu \nu}^{a b}(k)=-g_{\mu \nu}\left[\frac{p}{p_{1}} \delta^{a b}+\frac{2\left(k W^{a b}\right)}{p_{1}}+\left(\frac{1}{k^{2}}-\frac{p}{p_{1}}\right) \frac{\left(V^{a} V^{b}\right)}{V^{2}}\right] \tag{19}
\end{equation*}
$$

where the following notation has been introduced:

$$
\begin{equation*}
p(k, V)=k^{2}+g^{2} V^{2} \quad p_{1}(k, V)=p^{2}-4 g^{2}(k V)^{2} \tag{20}
\end{equation*}
$$

A solution of the ghost propagator equation (17) is also given by equation (19) (without $\left.g_{\mu \nu}\right)$.

Except for equations (14) and (17), the first-step equations give one more relation that contains a quantity $D_{\mu \nu}(0)$, which should be understood as some regularization. In essence this relation is a condition for the first-step counterterms $\delta z_{1}$. (There is no need to introduce the leading approximation counterterms in this case, i.e. $\delta z_{0}=0$ ). Since the Green functions of the first step are finite, this condition for the counterterms is necessary to remove the ultraviolet divergences at the second step of the scheme. This peculiarity of the given iteration scheme is displayed here in exactly the same manner as for the scalar field theory (see [2]).

## 4. Gauge field propagator

Let us now turn to a possible physical interpretation of the solutions. At $V_{\mu}=0 D_{\mu \nu}$ and $\Delta$ are free propagators of the gauge and ghost fields, and the whole iteration scheme is a reconstructed series of the perturbation theory. At $\boldsymbol{V}_{\mu} \neq 0$ the situation is more complicated. It is clear that in this case it is difficult to interpret the function $D_{\mu \nu}$ given by (19) as the propagator of a particle in the Poincare invariant theory. Let us recall, however, that we have a number $\{G(V)\}$ of solutions of the SDEs, each corresponding to some vector $\boldsymbol{V}_{\mu}$ satisfying the condition (18). Due to equation (18) the field strength tensor is identically zero and any such solution possesses zero energy. Therefore, a candidate for the 'physical vacuum functional' for this set of solutions is a superposition of all solutions. We shall exploit this fact in the construction of a Poincaré-invariant solution which can be interpreted as a particle propagator, i.e. a function depending only on the momentum $k$ and the scalar quantity

$$
\begin{equation*}
v^{2}=V^{2} \equiv V_{\mu}^{a} V_{\mu}^{a} \tag{21}
\end{equation*}
$$

(The quantity $v^{2}$ plays the role of an order parameter.) The construction is equivalent, in essence, to some averaging, i.e. an integration with a measure $\mathrm{d} \mu(V)$, we shall therefore denote it by angular brackets: $\langle G\rangle=\sum_{V} G(V)$, etc. In the foundation of this operation we set the following conditions:

$$
\begin{equation*}
\left\langle V_{\mu}^{a}\right\rangle=0 \quad\left\langle V^{2}\right\rangle=v^{2} \tag{22}
\end{equation*}
$$

Their necessity for the Poincaré-invariant theory is evident. It is also evident that

$$
\begin{equation*}
\left\langle V_{\mu}^{a} V_{\nu}^{b}\right\rangle=\frac{1}{4} v^{2} g_{\mu \nu} E^{a b} \tag{23}
\end{equation*}
$$

where $\operatorname{tr} E=1$. To determine the form of the matrix $E^{a b}$, consider the leading approximation condition (18). The geometrical meaning of condition (18) is the collinearity of the vectors $\boldsymbol{V}_{\mu}$ in isotopic space. Consequently, at $\boldsymbol{V}_{\mu} \neq 0$ there exists a selected direction in isotopic space. This direction can be chosen as a basis vector, for example $\boldsymbol{n}_{3}$. In this basis $V_{\mu}^{a}=\delta^{a 3} v_{\mu}$, and $E^{a b}=\delta^{a 3} \delta^{b 3}$. Therefore, at $\boldsymbol{V}_{\mu} \neq 0$ isotopic symmetry is spontaneously broken.

Further calculation is reduced to the averaging of the functions $f\left((k V)^{2}\right)=p / p_{1}$ and $f^{a b}=f \cdot\left(V^{a} V^{b}\right) / V^{2}$. First of all, note that at $k \rightarrow 0, f \rightarrow 1 / g^{2} v^{2}$, and at $k \rightarrow \infty, f \rightarrow 1 / k^{2}$. These Poincaré-invariant properties of $f$ should, of course, be conserved for $\langle f\rangle$ as well.

With equations (22)-(24) and its generalizations for an arbitrary monomial in $\boldsymbol{V}_{\mu}$, the following formulae can be proved:

$$
\begin{align*}
& \left\langle(k V)^{2 n}\right\rangle=\left(k^{2} v^{2}\right)^{n} \frac{\Gamma\left(n+\frac{1}{2}\right)}{(n+1)!\Gamma\left(\frac{1}{2}\right)}  \tag{24}\\
& \left\langle(k V)^{2 n}\left(V^{a} V^{b}\right)\right\rangle=E^{a b} v^{2}\left\langle(k V)^{2 n}\right\rangle
\end{align*}
$$

which are necessary for the calculation of $\left\langle D_{\mu \nu}\right\rangle$. The result of the calculation in the above basis is as follows:

$$
\begin{align*}
& \left\langle\tilde{D}_{\mu \nu}^{33}(k)\right\rangle=-g_{\mu \nu} \frac{1}{k^{2}}  \tag{25}\\
& \left\langle\tilde{D}_{\mu \nu}^{11}(k)\right\rangle=\left\langle\tilde{D}_{\mu \nu}^{22}(k)\right\rangle=-g_{\mu \nu} \frac{k^{2}+g^{2} v^{2}}{2 g^{2} v^{2} k^{2}}\left(1-\sqrt{1-\frac{4 g^{2} v^{2} k^{2}}{\left(k^{2}+g^{2} v^{2}\right)^{2}}}\right) \tag{26}
\end{align*}
$$

(Other isotopic components are equal to zero.) Therefore, along the selected isotopic direction the particle propagates as a free one, but along other directions a separation of the region of momenta exists, the scale of the separation being the quantity $g^{2} v^{2}$. It is necessary to stress that both the limiting cases above ( $k^{2} \rightarrow 0$ and $k^{2} \rightarrow \infty$ ) belong to the region of applicability of the calculations performed, which is defined by the condition $\left|4 g^{2} v^{2} k^{2} /\left(k^{2}+g^{2} v^{2}\right)^{2}\right|<1$. Consider the question of analytical continuation. Equation (26) defines two analytical functions depending on the choice of branch of the function $\sqrt{z^{2}}$, but neither satisfies both the above asymptotic conditions simultaneously and, consequently, these functions are not solutions of the problem. Hence, as a solution one should choose at large $k$ the branch with the behaviour $1 / k^{2}$, and at small $k$ another branch which is a constant $1 / g^{2} v^{2}$. At the points $\pm g^{2} v^{2}$ the solution goes from one branch to another, i.e. from the 'perturbative' sheet to the 'non-perturbative' one. At the point $k^{2}=g^{2} v^{2}$ the solution is continuous, and at the point $k^{2}=-g^{2} v^{2}$ it has a discontinuity. (Note that the first-step calculations of the iteration scheme do not fix the sign of $v^{2}$.) Such unusual features of the solution near the separation points are likely to be connected with our limitation of the characteristic equation solutions. Probably an extension of the class of solutions will lead to smoothing of the propagator behaviour near the points of separation of the perturbative and non-perturbative regions.

## 5. Discussion

In this paper we have attempted to introduce a new iteration scheme to solve the SchwingerDyson equations for non-Abelian gauge theory. As a very first step we have considered a scheme based on the simplest constant solutions of the characteristic equations (5). Of course, there are no prior grounds for expecting such a set of solutions to be useful for physical applications. Some other sets of more non-trivial solutions (instanton-like and so on) of the characteristic equations are likely to be of real physical interest. Dealing with coordinate-dependent solutions is a more difficult task; in particular, the averaging procedure of section 4 should be essentially modified. Another important question is the correspondence of the proposed method with the functional integral approaches, such as the well known background field method [6]. In spite of their evident similarity, the proposed iteration scheme seems to be somewhat different: the structure of the expansions is distinct. The problem of the correspondence requires further investigations based on the calculation of higher steps of the iteration scheme.

The set of quasiperturbative solutions of sections 3 and 4 leads to spontaneous breaking of $S U(2)$ gauge symmetry. The question arises as to whether such solutions can be used to construct a realistic model of electroweak interactions in the spirit of dynamical gauge symmetry breaking models without Higgs bosons (see [7] and references therein). Further investigation which should essentially extend the class of solutions is necessary to eliminate this question. On the other hand, the set of quasiperturbative solutions satisfying condition (18) considered here does not exhaust the solutions with zero energy, and some further averaging to restore the symmetry may be necessary.

For $S U(3)$ gauge group (the case of QCD ) the variety of solutions increases greatly in comparison with the $S U(2)$ group (see, for example, [8]). Avoiding discussion of this case, nevertheless note that the infrared finite behaviour of the gluon field propagator has recently been discussed in detail (see [9] and references therein).

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